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Production of massless fermions during inflation

Tomislav Prokopec

Institut für Theoretische Physik, Heidelberg University
Philosophenweg 16, D-69120 Heidelberg, Germany and
Theory Division, CERN, CH-1211 Geneva 23, Switzerland
E-mail: T.Prokopec@thphys.uni-heidelberg.de, Tomislav.Prokopec@cern.ch

Richard Paul Woodard

Department of Physics, University of Florida
Gainesville, Florida 32611, U.S.A.
E-mail: woodard@phys.ufl.edu

Abstract: We compute the one loop self energy, in a locally de Sitter background, for a massless fermion which is Yukawa-coupled to a massless, minimally coupled scalar. We then solve the modified Dirac equation resulting from inclusion of the self energy. We find faster-than-exponential growth in the fermion wave function, consistent with the production of fermions through a process in which a scalar and a fermion-anti-fermion pair are ripped out of the vacuum by inflation.

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1. Introduction

Consider quantum field theory in the presence of a homogeneous, isotropic and spatially flat geometry,

$$ds^2 = -dt^2 + a^2(t) d\vec{x} \cdot d\vec{x} \,. \tag{1.1}$$

By virtue of spatial translation invariance we can label particle states by their constant wave vectors, $\vec{k}$. However, the physical 3-momentum associated with this label is $\vec{k}/a(t)$. And if such a particle has mass m, its energy is,

$$E(t, \vec{k}) = \sqrt{m^2 + \frac{\|\vec{k}\|^2}{a^2(t)}} \,. \tag{1.2}$$

Now consider the appearance, at time t, of a pair of virtual particles with wave vectors $\pm\vec{k}$. This process conserves 3-momentum but not energy. According to the energy-time uncertainty principle this violation of energy conservation will not be detectable provided the pair disappears back into the vacuum before a time interval Δt such that,

$$\int_t^{t+\Delta t} dt' \, 2E(t', \vec{k}) \lesssim 1 \,. \tag{1.3}$$

For any $m \neq 0$ the integral grows without bound, so Δt is finite for massive particles. However, if $m = 0$ and the scale factor grows rapidly enough, the integral remains bounded even for $\Delta t \longrightarrow \infty$. This means that massless virtual particles need never recombine.

"Rapidly enough" turns out to mean $\ddot{a}(t) > 0$, which is a type of expansion known as "inflation" [1]. Whether or not inflation leads to a significant amount of particle production for a given species depends upon the rate at which virtual particles of that species emerge from the vacuum. It turns out that the emergence rate for particles which are conformally

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invariant falls like $1/a(t)$ [2]. This means that any conformally invariant and massless virtual particles which happen to emerge from the vacuum will become real, but not many emerge. Hence there is negligible direct production of conformally invariant particles during inflation.

Most familiar massless particles possess conformal invariance. In particular, photons are conformally invariant in $D = 4$ spacetime dimensions, and massless Dirac fermions are conformally invariant in any dimension. On the other hand, neither massless, minimally coupled scalars nor gravitons are conformally invariant, and both are produced copiously during inflation. This effect is observable; the production of nearly massless, minimally coupled scalars is believed to be responsible for the primordial anisotropies in the cosmic ray microwave background [3, 4].

One naturally wonders what happens when a massless but conformally invariant particle interacts with a massless particle which is not conformally invariant. The first step in answering this question is to compute the one particle irreducible (1PI) 2-point function of the conformally invariant field. One next uses this 1PI 2-point function to correct the linearized equations of motion. The effect of inflation on the conformally invariant particle can be inferred by solving these equations for the plane wave mode functions.

We have previously carried out such a study for photons interacting with a charged, massless and minimally coupled scalar [5, 6]. The result is that the mode functions of super-horizon wave vectors oscillate around a nonzero constant with increasing frequency and decreasing amplitude [7]. This implies zero photon production and an enormous enhancement in the 0-point energy, roughly similar to what happens for massive photons in Proca electrodynamics. The physical interpretation is that the inflationary production of charged scalars results in a diffuse plasma which inhibits the propagation of light.

The purpose of this paper is to study what happens when Dirac fermions are made to Yukawa-interact with a real, massless and minimally coupled scalar. We will show that although the fermion mass remains zero, the mode functions for super-horizon wave vectors grow in a manner that is consistent with the production of fermions. The physical interpretation seems to be that inflation alters the constraint of energy conservation to permit the spontaneous appearance of a scalar and a fermion-anti-fermion pair.

In section 2 we compute the one loop fermion self energy. In section 3 this is used to correct the Dirac equation. Although this equation is nonlocal, an asymptotic form is derived for the late time behavior of a spatial plane wave. In section 4 we show that the particle production inferred from the mode functions is consistent with a 3-particle creation process. Our results are summarized and discussed in section 5.

2. The one loop self energy

We begin by reviewing the conventions appropriate to Dirac fields in a nontrivial geometry. In order to facilitate dimensional regularization we make no assumption about the spacetime dimension D. The gamma matrices γ^b_{ij} ($b = 0, 1, \ldots, D-1$) anti-commute in the usual way, $\{\gamma^b, \gamma^c\} = -2\eta^{bc} I$. One interpolates between local Lorentz indices $(b, c, d, \ldots)$ and vector indices (lower case Greek letters) with the vierbein field, $e_{\mu b}(x)$. The metric is ob-

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tained by contracting two vierbeins with the Minkowski metric, $g_{\mu\nu}(x) = e_{\mu b}(x)e_{\nu c}(x)\eta^{bc}$. The vierbein's vector index is raised and lowered by the metric ($e^{\mu}_{\ b} = g^{\mu\nu}e_{\nu b}$) while the local Lorentz index is raised and lowered with the Minkowski metric ($e_{\mu}^{\ b} = \eta^{bc}e_{\mu c}$). The spin connection and the Lorentz representation matrices are,

$$A_{\mu bc} \equiv e^{\nu}_{\ b}\Bigl(e_{\nu c,\mu} - \Gamma^{\rho}_{\ \mu\nu}e_{\rho c}\Bigr) , \qquad J^{bc} \equiv \frac{i}{4}[\gamma^b, \gamma^c] . \tag{2.1}$$

Let $\phi(x)$ represent a real scalar field and let $\psi_i(x)$ stand for a Dirac field. In a general background metric the lagrangian we wish to study would be,

$$\mathcal{L}_{\rm gen} = -\frac{1}{2}\partial_{\mu}\phi\partial_{\nu}\phi g^{\mu\nu}\sqrt{-g} + \overline{\psi}e^{\mu}_{\ b}\gamma^b\Bigl(i\partial_{\mu} - \frac{1}{2}A_{\mu cd}J^{cd}\Bigr)\psi\sqrt{-g} - f\phi\overline{\psi}\psi\sqrt{-g} , \tag{2.2}$$

where $\overline{\psi} \equiv \psi^{\dagger}\gamma^0$ is the usual Dirac adjoint and f is the Yukawa coupling constant. The geometry of interest is the very special form associated with the homogeneous and isotropic element (1.1). By defining a new time coordinate $d\eta \equiv dt/a(t)$, the metric of this geometry can be made conformal to the Minkowski metric,

$$ds^2 = a^2\Bigl(-d\eta^2 + d\vec{x}\cdot d\vec{x}\Bigr) . \tag{2.3}$$

A convenient choice for the associated vierbein is, $e_{\mu b} = a\eta_{\mu b}$. With these simplifications the spin connection assumes the form,

$$e_{\mu b} = a\eta_{\mu b} \Longrightarrow A_{\mu cd} = \Bigl(\eta_{\mu c}\partial_d - \eta_{\mu d}\partial_c\Bigr)\ln(a) . \tag{2.4}$$

And our lagrangian reduces to,

$$\mathcal{L}_{\rm conf} = -\frac{1}{2}a^{D-2}\partial_{\mu}\phi\partial_{\nu}\phi\eta^{\mu\nu} + \Bigl(a^{\frac{D-1}{2}}\overline{\psi}\Bigr)i\gamma^{\mu}\partial_{\mu}\Bigl(a^{\frac{D-1}{2}}\psi\Bigr) - fa^D\phi\overline{\psi}\psi . \tag{2.5}$$

Having defined the lagrangian we now give the propagators and vertices which comprise the position space Feynman rules. Of course vertices are always straightforward. The $\phi\overline{\psi}_i\psi_j$ vertex and the fermion field strength renormalization are,

$$-ifa^D\delta_{ij} \qquad \text{and} \qquad i\delta Z_2(aa')^{\frac{D-1}{2}}\gamma^{\mu}_{ij}\,i\partial_{\mu}\delta^D(x-x') , \tag{2.6}$$

where $a \equiv a(t)$ and $a' \equiv a(t')$ are the scale factors evaluated at $x^0 = \eta$ and $x'^0 = \eta'$, respectively. We do not need scalar mass and field strength renormalization.

The fermion propagator is best expressed in terms of the conformal coordinate interval,

$$\Delta x^2(x;x') \equiv \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2 . \tag{2.7}$$

Note that we label the position in spacetime through the D-vector $x^{\mu} = (\eta, \vec{x})$. We see from the second term of (2.5) that, for $f = 0$, the combination $a^{\frac{D-1}{2}}\psi$ behaves like a free, massless Dirac field in flat space. It follows that the propagator of ψ is just a conformal rescaling of the flat space result,

$$\begin{aligned} i[_iS_j](x;x') &= (aa')^{\frac{1-D}{2}}\gamma^{\mu}_{ij}\,i\partial_{\mu}\left\{\frac{\Gamma(\frac{D}{2}-1)}{4\pi^{\frac{D}{2}}}\Bigl[\Delta x^2(x;x')\Bigr]^{1-\frac{D}{2}}\right\} \\ &= \frac{\Gamma(\frac{D}{2})}{2\pi^{\frac{D}{2}}}\frac{(aa')^{\frac{1-D}{2}}(-i)\gamma^{\mu}_{ij}\Delta x_{\mu}}{[\Delta x^2(x;x')]^{\frac{D}{2}}} . \end{aligned} \tag{2.8}$$

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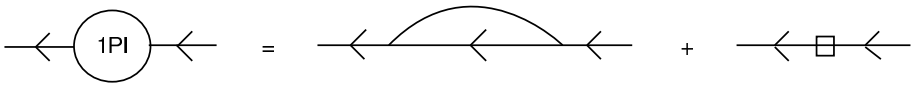

Figure 1: The one loop fermion self energy. Fermion lines have an arrow, scalar lines do not. The final diagram gives the contribution of field strength renormalization.

Here $\Delta x_\mu \equiv \eta_{\mu\nu}(x^\nu - x'^\nu)$. The split index notation in $i[{}_iS_j](x;x')$ indicates that the first index (i) transforms according to the local Lorentz group at the first coordinate argument (x^μ) whereas the second index (j) transforms at the second argument (x'^μ).

It has not been necessary so far to make any assumption about the form of the scale factor. However, the free scalar is not conformally invariant, which makes its plane wave mode functions depend upon $a(t)$ in a complicated manner. Although the solution for general $a(t)$ has recently been obtained [8], the Fourier mode sum has only been worked out for certain cases. We therefore specialize to locally de Sitter inflation,

$$a(t) = e^{Ht} = -\frac{1}{H\eta}\,. \tag{2.9}$$

In this geometry there is a simple relation between $\ell(x;x')$, the invariant length from x^μ to x'^μ, and the conformal coordinate interval $\Delta x^2(x;x')$,

$$4\sin^2\left(\frac{1}{2}H\ell(x;x')\right) = aa'H^2\Delta x^2(x;x') \equiv y(x;x')\,. \tag{2.10}$$

We refer to $y(x;x')$ as the de Sitter length function. The scalar propagator can be expressed in terms of $y \equiv y(x;x')$ and the two scale factors,

$$\begin{aligned} i\Delta(x;x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\Bigg\{ & 2^{D-4}\Gamma(D-1)\ln(aa') - \pi\cot(\pi\tfrac{D}{2})\frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} + \\ & + \sum_{n=1}^{\infty}\Bigg[\frac{1}{n}\frac{\Gamma(D-1+n)}{\Gamma(\frac{D}{2}+n)}\Big(\frac{y}{4}\Big)^n - \frac{1}{n-\frac{D}{2}}\frac{\Gamma(\frac{D}{2}-1+n)}{\Gamma(n)}\Big(\frac{y}{4}\Big)^{n-\frac{D}{2}}\Bigg]\Bigg\}\,. \end{aligned} \tag{2.11}$$

Note that the homogeneous terms (i.e., y^n) and the constant terms have slightly different proportionality constants from our previous expression [9, 6]. This has been done to make (2.11) valid for regulating a spacetime in which D will ultimately be taken to some dimension other than four [1]. In $D = 4$, the propagator (2.11) reduces to a more familiar, elementary form,

$$i\Delta(x;x') \stackrel{D\rightarrow 4}{\longrightarrow} \frac{H^2}{4\pi^2}\Bigg\{\frac{\eta\eta'}{\Delta x^2} - \frac{1}{2}\ln(H^2\Delta x^2) - \frac{1}{4} + \ln(2)\Bigg\}\,. \tag{2.12}$$

The diagrammatic representation of the one loop fermion self energy is given in figure 1. Computing it is simple: just multiply the various vertices and propagators,

$$\begin{aligned} -i\Big[{}_i\Sigma_j\Big](x;x') = & \Big(-ifa^D\delta_{ik}\Big)i\Big[{}_kS_\ell\Big](x;x')\Big(-ifa'^D\delta_{\ell j}\Big)i\Delta(x;x') + \\ & + i\delta Z_2(aa')^{\frac{D-1}{2}}\gamma^\mu_{ij}\,i\partial_\mu\delta^D(x-x')\,. \end{aligned} \tag{2.13}$$

[1] We thank Ewald Puchwein for pointing out the improvement.

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To renormalize in four dimensions, set $D = 4 - \epsilon$ and then expand in ϵ. Only terms which are not integrable over d^4x' need to have ϵ arbitrary,

$$-f^2(aa')^D i\Big[{}_iS_j\Big](x;x')i\Delta(x;x') = \frac{f^2(aa')^{\frac{3}{2}}}{2^3\pi^{4-\epsilon}}\Gamma\Big(2-\frac{\epsilon}{2}\Big)\Gamma\Big(1-\frac{\epsilon}{2}\Big)\frac{i\gamma^\mu_{ij}\Delta x_\mu}{(\Delta x^2)^{3-\epsilon}} - \\ -\frac{f^2H^2(aa')^{\frac{5}{2}}}{2^4\pi^4}\ln\Big(\frac{\sqrt{e}}{4}H^2\Delta x^2\Big)\frac{i\gamma^\mu_{ij}\Delta x_\mu}{(\Delta x^2)^2} + O(\epsilon)\,. \quad (2.14)$$

The next step is writing the factor of Δx_μ as a derivative,

$$\Gamma\Big(2-\frac{\epsilon}{2}\Big)\Gamma\Big(1-\frac{\epsilon}{2}\Big)\frac{i\gamma^\mu_{ij}\Delta x_\mu}{(\Delta x^2)^{3-\epsilon}} = -\frac{1}{4}\Gamma^2\Big(1-\frac{\epsilon}{2}\Big)i\gamma^\mu_{ij}\partial_\mu\left(\frac{1}{\Delta x^2}\right)^{2-\epsilon}, \quad (2.15)$$

$$\ln\Big(\frac{\sqrt{e}}{4}H^2\Delta x^2\Big)\frac{i\gamma^\mu_{ij}\Delta x_\mu}{(\Delta x^2)^2} = -\frac{1}{2}i\gamma^\mu_{ij}\partial_\mu\left(\frac{\ln\Big(\frac{\sqrt{e}}{4}H^2\Delta x^2\Big)+1}{\Delta x^2}\right). \quad (2.16)$$

One then expresses inverse powers of Δx^2 as powers of $\partial^2 \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$ acting on logarithms,

$$\frac{\ln\Big(\frac{\sqrt{e}}{4}H^2\Delta x^2\Big)+1}{\Delta x^2} = \frac{1}{8}\partial^2\ln^2\Big(\frac{\sqrt{e}}{4}H^2\Delta x^2\Big)\,. \quad (2.17)$$

It is at this stage that the renormalization scale μ appears and that the ultraviolet divergence is segregated to a local term,

$$\left(\frac{1}{\Delta x^2}\right)^{2-\epsilon} = -\frac{1}{32}\partial^4\Big[\ln^2(\mu^2\Delta x^2) - 2\ln(\mu^2\Delta x^2)\Big] - \frac{2i\pi^{2-\frac{\epsilon}{2}}\mu^{-\epsilon}}{\epsilon(1-\epsilon)\Gamma(1-\frac{\epsilon}{2})}\delta^D(x-x') + O(\epsilon)\,. \quad (2.18)$$

Defining $\gamma^\mu_{ij}\partial_\mu \equiv \not{\partial}_{ij}$ and combining terms we have,

$$-f^2(aa')^D i\Big[{}_iS_j\Big](x;x')i\Delta(x;x') = \frac{f^2(aa')^{\frac{3}{2}}}{2^{10}\pi^4}\,i\not{\partial}_{ij}\partial^4\Big[\ln^2(\mu^2\Delta x^2) - 2\ln(\mu^2\Delta x^2)\Big] + \\ +\frac{f^2H^2(aa')^{\frac{5}{2}}}{2^8\pi^4}\,i\not{\partial}_{ij}\partial^2\ln^2\Big(\frac{\sqrt{e}}{4}H^2\Delta x^2\Big) + \quad (2.19) \\ +\frac{if^2\mu^{-\epsilon}(aa')^{\frac{3}{2}}}{2^4\pi^{2-\frac{\epsilon}{2}}}\frac{\Gamma(1-\frac{\epsilon}{2})}{\epsilon(1-\epsilon)}\,i\not{\partial}_{ij}\delta^D(x-x') + O(\epsilon)\,.$$

The simplest renormalization condition is,

$$\delta Z_2 = -\frac{f^2\mu^{-\epsilon}}{2^4\pi^{2-\frac{\epsilon}{2}}}\frac{\Gamma(1-\frac{\epsilon}{2})}{\epsilon(1-\epsilon)}\,. \quad (2.20)$$

We obtain the renormalized self energy by taking the limit $\epsilon \to 0$,

$$\Big[{}_i\Sigma^{\rm ren}_j\Big](x;x') = -\frac{f^2(aa')^{\frac{3}{2}}}{2^{10}\pi^4}\,\not{\partial}_{ij}\partial^4\Big[\ln^2(\mu^2\Delta x^2) - 2\ln(\mu^2\Delta x^2)\Big] - \\ -\frac{f^2(aa')^{\frac{3}{2}}}{2^5\pi^2}\ln(aa')\,i\not{\partial}_{ij}\delta^4(x-x') - \\ -\frac{f^2H^2(aa')^{\frac{5}{2}}}{2^8\pi^4}\,\not{\partial}_{ij}\partial^2\ln^2\Big(\frac{\sqrt{e}}{4}H^2\Delta x^2\Big) + O(f^4)\,. \quad (2.21)$$

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The first term is the conformally rescaled flat space result. The second term is the well-known contribution from the conformal anomaly. The intrinsic de Sitter result is the third term. It is distinguished from the other two by its extra factor of aa'.

3. Solving the modified Dirac equation

The self energy modifies the Dirac equation to include quantum corrections. The modified equation takes the form,

$$a^{\frac{3}{2}} i \not{\partial}_{ij} \Bigl(a^{\frac{3}{2}} \psi_i(x) \Bigr) + \int d^4x' \Bigl[{}_i\Sigma_j \Bigr](x;x') \psi_j(x') = 0 \,. \tag{3.1}$$

It describes how the mode functions change due to quantum corrections. From these mode functions one can infer particle production, and also kinematical properties such as mass.

There is some ambiguity about what it means to "include quantum corrections," and each of the possible meanings corresponds to employing a different self energy functional in the modified Dirac equation (3.1). The in-out self energy (2.21) corresponds to a particle which is surrounded by free vacuum in the distant past, and for which quantum effects conspire to leave it surrounded by free vacuum in the distant future. This self energy is appropriate to flat space scattering experiments, but it is not very realistic for cosmological settings. A better choice for cosmology is to release the particle surrounded by free vacuum at some finite time and then let it evolve as it will. For that situation it is the Schwinger-Keldysh self energy which is appropriate [10].

At one loop order the Schwinger-Keldysh self energy is simple to construct from the in-out self energy (2.21). We will give the technique without deriving it [11]. Each vertex point is endowed with a polarity $\pm$. In place of the conformal interval (2.7) we define,

$$\Delta x^2_{++} \equiv \Delta r^2 - (|\Delta\eta| - i\delta)^2 \,, \qquad \Delta x^2_{+-} \equiv \Delta r^2 - (\Delta\eta + i\delta)^2 \,, \tag{3.2}$$

where $\Delta r \equiv \|\vec{x} - \vec{x}'\|$ and $\Delta\eta \equiv \eta - \eta'$. At one loop order we do not require the $-+$ or $--$ variations. The $++$ and $+-$ self energies are,

$$\begin{aligned} \Bigl[{}_i\Sigma^{\rm ren}_j \Bigr](x_+;x'_+) &= -\frac{f^2(aa')^{\frac{3}{2}}}{2^{10}\pi^4} \not{\partial}_{ij} \partial^4 \Bigl[\ln^2(\mu^2 \Delta x^2_{++}) - 2\ln(\mu^2 \Delta x^2_{++}) \Bigr] - \\ &\quad - \frac{f^2(aa')^{\frac{3}{2}}}{2^5\pi^2} \ln(aa')\, i\not{\partial}_{ij} \delta^4(x-x') - \\ &\quad - \frac{f^2 H^2 (aa')^{\frac{5}{2}}}{2^8\pi^4} \not{\partial}_{ij} \partial^2 \ln^2\Bigl(\frac{\sqrt{e}}{4} H^2 \Delta x^2_{++} \Bigr) + O(f^4) \,, \end{aligned} \tag{3.3}$$

$$\begin{aligned} \Bigl[{}_i\Sigma^{\rm ren}_j \Bigr](x_+;x'_-) &= +\frac{f^2(aa')^{\frac{3}{2}}}{2^{10}\pi^4} \not{\partial}_{ij} \partial^4 \Bigl[\ln^2(\mu^2 \Delta x^2_{+-}) - 2\ln(\mu^2 \Delta x^2_{+-}) \Bigr] + \\ &\quad + \frac{f^2 H^2 (aa')^{\frac{5}{2}}}{2^8\pi^4} \not{\partial}_{ij} \partial^2 \ln^2\Bigl(\frac{\sqrt{e}}{4} H^2 \Delta x^2_{+-} \Bigr) + O(f^4) \,. \end{aligned} \tag{3.4}$$

The Schwinger-Keldysh self energy is the sum of these,

$$\Bigl[{}_i\Sigma^{\rm SK}_j \Bigr](x;x') \equiv \Bigl[{}_i\Sigma^{\rm ren}_j \Bigr](x_+;x'_+) + \Bigl[{}_i\Sigma^{\rm ren}_j \Bigr](x_+;x'_-) \,. \tag{3.5}$$

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Our explicit result for $[{}_i\Sigma^{\rm SK}_j](x;x')$ is,

$$-\frac{f^2(aa')^{\frac{3}{2}}}{2^8\pi^3}\,i\not{\partial}_{ij}\partial^4\Big\{\theta(\Delta\eta)\theta(\Delta\eta-\Delta r)\ln\Big[e^{-1}\mu^2(\Delta\eta^2-\Delta r^2)\Big]\Big\}-$$
$$-\frac{f^2(aa')^{\frac{3}{2}}}{2^5\pi^2}\ln(aa')\,i\not{\partial}_{ij}\delta^4(x-x')-$$
$$-\frac{f^2H^2(aa')^{\frac{5}{2}}}{2^6\pi^3}\,i\not{\partial}_{ij}\partial^2\Big\{\theta(\Delta\eta)\theta(\Delta\eta-\Delta r)\ln\Big[\frac{\sqrt{e}}{4}H^2(\Delta\eta^2-\Delta r^2)\Big]\Big\}+O(f^4)\,. \tag{3.6}$$

Note that it vanishes whenever the point x'^μ is outside the past light-cone of x^μ. This means that although the modified Dirac equation (3.1) is nonlocal, it is nonetheless causal. Another important property of $[{}_i\Sigma^{\rm SK}_j](x;x')$ is that each term contains only a single gamma matrix. This means that it conserves helicity, and hence that the fermion remains massless.

Note also that the first term of (3.6), the flat space part, has the same number of scale factors as the tree order differential operator in (3.1). Perturbation theory only makes sense if the coupling constant is small, so we may assume $f^2 \ll 1$. This means that the flat space part of the one loop self energy can be ignored. So too can the conformal anomaly, which is only enhanced by a factor of $\ln(a)$. However, we cannot necessarily ignore the de Sitter correction. Although it is also down by f^2, it is enhanced by a scale factor, which becomes enormously large during inflation. It turns out that higher loop corrections — which are down by more powers of f — can never give more than this one factor of a. It therefore makes sense to keep only the one loop de Sitter contribution of (3.6),

$$\Big[{}_i\Sigma_j\Big](x;x') \longrightarrow -\frac{f^2H^2(aa')^{\frac{5}{2}}}{2^6\pi^3}\,i\not{\partial}_{ij}\partial^2\Big\{\theta(\Delta\eta)\theta(\Delta\eta-\Delta r)\ln\Big[\frac{\sqrt{e}}{4}H^2(\Delta\eta^2-\Delta r^2)\Big]\Big\}\,. \tag{3.7}$$

This is the "self energy" we will henceforth employ in the modified Dirac equation (3.1).

Because the background is spatially translation invariant it makes sense to look for plane wave solutions,

$$\psi_i(\eta,\vec{x}) = a^{-\frac{3}{2}}\chi_i(\eta,k)e^{i\vec{k}\cdot\vec{x}}\,, \tag{3.8}$$

To take advantage of helicity conservation we employ chiral gamma matrices,

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{3.9}$$

and we factor out the tree η dependence of the 2-component helicity eigenstates,

$$\chi_i(\eta,k) = \begin{pmatrix} \chi^\pm_L(\eta,k)e^{\pm ik\eta} \\ \chi^\pm_R(\eta,k)e^{\mp ik\eta} \end{pmatrix} \qquad \text{with} \qquad \vec{k}\cdot\vec{\sigma}\,\chi^\pm_{L,R} = \pm k\,\chi^\pm_{L,R}\,. \tag{3.10}$$

For the left-handed spinor the result of performing the spatial integrations is,

$$\partial_0\chi^\pm_L(\eta,k) + \frac{f^2H^2}{4\pi^2}a\int_{\eta_i}^{\eta}d\eta' a'\chi^\pm_L(\eta',k)e^{\mp i2k\Delta\eta}\left[\ln(H\Delta\eta)+\frac{3}{4}+\frac{1}{2}\int_0^{2k\Delta\eta}d\tau\frac{e^{\pm i\tau}-1}{\tau}\right] = 0\,. \tag{3.11}$$

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Here $\eta_i = -1/H$ is the initial conformal time, which corresponds to $t = 0$. The analogous equation for the right-handed spinor with $\pm$ helicity is,

$$\partial_0 \chi_R^{\pm}(\eta,k) + \frac{f^2H^2}{8\pi^2} a \int_{\eta_i}^{\eta} d\eta' a' \chi_R^{\pm}(\eta',k) e^{\pm i2k\Delta\eta} \left[2\ln(H\Delta\eta) + \frac{3}{2} + \int_0^{2k\Delta\eta} d\tau \frac{e^{\mp i\tau}-1}{\tau} \right] = 0 \,. \tag{3.12}$$

Recovering the full time evolution of nonlocal equations such as (3.11-3.12) requires numerical integration. However, it is straightforward to get the asymptotic behavior for late times. First change the independent variable to the number of e-foldings, $N \equiv \ln(a)$. Now redefine the dependent variable as $h_{L,R}^{\pm}(N,w) \equiv \chi_{L,R}^{\pm}(-H^{-1}e^{-N}, Hw)$. The resulting equations are,

$$\begin{aligned} \partial_N h_L^{\pm}(N,w) + \Bigl(\frac{f}{2\pi}\Bigr)^2 \int_0^N dN' \, h_L^{\pm}(N',w) \exp\Bigl[\mp i2w(e^{-N'}-e^{-N})\Bigr] \times \\ \times \Biggl\{ \ln\Bigl[e^{-N'}-e^{-N}\Bigr] + \frac{3}{4} + \frac{1}{2} \int_0^{2w(e^{-N'}-e^{-N})} d\tau \, \frac{e^{\pm i\tau}-1}{\tau} \Biggr\} = 0 \,. \end{aligned} \tag{3.13}$$

The equations for the right-handed spinor are obtained by replacing $\mp i2w$ with $\pm 2iw$ and $\pm i\tau$ with $\mp i\tau$.

To obtain the asymptotic form for large-N we simplify (3.13) by dropping terms in the integrand which are exponentially small in N for fixed $N' < N$,

$$\partial_N h_L^{\pm}(N,w) + \Bigl(\frac{f}{2\pi}\Bigr)^2 \int_0^N dN' \, h_L^{\pm}(N',w) e^{\mp i2we^{-N'}} \left[-N' + \frac{3}{4} + \frac{1}{2} \int_0^{2we^{-N'}} d\tau \, \frac{e^{\pm i\tau}-1}{\tau} \right] \approx 0 \,. \tag{3.14}$$

At large-N we see that the system has the form,

$$\partial_N h_\infty(N,w) = \int_0^N dN' g_\infty(N',w) h_\infty(N',w) \,. \tag{3.15}$$

where the function $g_\infty(N,w)$ is,

$$\begin{aligned} g_\infty(N,w) = \Bigl(\frac{f}{2\pi}\Bigr)^2 \Biggl\{ N - \frac{3}{4} - \frac{1}{2} \int_0^{2we^{-N}} d\tau \, \frac{e^{\pm i\tau}-1}{\tau} \Biggr\} e^{\mp i2we^{-N}} \longrightarrow \\ \longrightarrow \Bigl(\frac{f}{2\pi}\Bigr)^2 \Bigl\{ N - \frac{3}{4} + O\Bigl(Ne^{-N}\Bigr) \Bigr\} \,. \end{aligned} \tag{3.16}$$

Because the function $g_\infty(N,w)$ grows, the integral is dominated by its upper limit and we can obtain the asymptotic form of $h_\infty(N,w)$ (up to a proportionality constant) using the WKB technique,

$$h_\infty(N,w) \longrightarrow g_\infty^{-\frac{1}{4}}(N,w) \exp\left[\int^N dN' \sqrt{g_\infty(N',w)} \right] \longrightarrow \Bigl(N - \frac{3}{4}\Bigr)^{-\frac{1}{4}} \exp\left[\frac{f}{3\pi} \Bigl(N - \frac{3}{4}\Bigr)^{\frac{3}{2}} \right] . \tag{3.17}$$

Each of the various helicities and polarizations, $h_L^{\pm}(N,w)$ and $h_R^{\pm}(N,w)$, approaches a constant times this same limiting form $h_\infty(N,w)$.

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4. The physics behind our result

4.1 Massless minimally coupled scalars

Most familiar particles become (classically) conformally invariant when their masses are taken to zero. The exceptions are gravitons and massless, minimally coupled scalars. The lagrangian density for the later is,

$$\mathcal{L}_{\rm MMCS} = -\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu}\sqrt{-g} = -\frac{1}{2}a^2\partial_\mu\phi\partial_\nu\phi\eta^{\mu\nu}\,. \tag{4.1}$$

The lagrangian is the spatial integral of the lagrangian density. Using Parseval's theorem to convert to Fourier space, and expressing the field in terms of co-moving (rather than conformal) time, gives a form we can recognize,

$$L_{\rm MMCS} \equiv \int d^3x \mathcal{L}_{\rm MMCS} = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\Big\{a^3(t)|\dot{\widetilde{\phi}}(t,\vec{p})|^2 - a(t)\|\vec{p}\|^2|\widetilde{\phi}(t,\vec{p})|^2\Big\}. \tag{4.2}$$

Each wave vector $\vec{p}$ represents an independent harmonic oscillator with a time dependent mass, $m(t) \sim a^3(t)$, and frequency, $\omega(t) = \|\vec{p}\|/a(t)$.

For the case of de Sitter inflation ($a(t) = e^{Ht}$) the time dependence of the part of the field with wave vector $\vec{k}$ takes the simple form,

$$\widetilde{\phi}(t,\vec{p}) = f(t,p)\alpha(\vec{p}) + f^*(t,p)\alpha^\dagger(-\vec{p}) \qquad \text{where} \qquad f(t,p) \equiv \frac{H}{\sqrt{2p^3}}\Big(1 - \frac{ip}{aH}\Big)e^{ip/Ha}\,. \tag{4.3}$$

To understand the meaning of the canonically normalized creation and annihilation operators operators $\alpha(\vec{p})$ and $\alpha^\dagger(\vec{p})$, note that the minimum energy in wave vector $\vec{p}$ at time t is $\frac{1}{2}(\hbar)\omega(t)$. However, the state with this energy does not evolve onto itself. Indeed, this theory has no stationary states! A reasonable "vacuum" is the state that was minimum energy in the distant past. This is known as Bunch-Davies vacuum and it is defined by $\alpha(\vec{p})|\Omega\rangle = 0$.

For Bunch-Davies vacuum the 0-point energy in wave vector $\vec{p}$ is,

$$E_0(t,\vec{p}) = \frac{1}{2}a^3(t)|\dot{f}(t,p)|^2 + \frac{1}{2}a(t)\|\vec{p}\|^2|f(t,p)|^2 = \frac{\|\vec{p}\|}{2a(t)} + \frac{H^2a(t)}{4\|\vec{p}\|}\,. \tag{4.4}$$

The first term on the far right represents the irreducible, minimum energy. The second term gives the extra energy due to inflationary particle production. Since the energy of a particle of wave number $\vec{p}$ is $\|\vec{p}\|/a(t)$, we can easily compute the number of particles,

$$N(t,\vec{p}) = \left(\frac{Ha(t)}{2\|\vec{p}\|}\right)^2. \tag{4.5}$$

As one might expect, this is much less than one at very early times, and it becomes of order one when the wave number just begins to satisfy the uncertainty bound (1.3).

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4.2 Production of fermions

Although we have seen that inflation produces enormous numbers of massless, minimally coupled scalars, the conformal invariance of free Dirac theory implies that there can be no comparable, direct production of fermions. However, it is still possible to make fermions during inflation by allowing them to interact with a massless, minimally coupled scalar. To see this we begin by giving the free field expansions of the interaction picture fields,

$$\phi_I(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \left\{ \alpha(\vec{p}) f(t,p) e^{i\vec{p}\cdot\vec{x}} + \alpha^\dagger(\vec{p}) f^*(t,p) e^{-i\vec{p}\cdot\vec{x}} \right\}, \tag{4.6}$$

$$\psi_I(t,\vec{x}) = \int \frac{d^3q}{(2\pi)^3} \sum_r \left\{ \beta(\vec{q},r) \frac{u(\vec{q},r)}{\sqrt{2q}} e^{iq/Ha + i\vec{q}\cdot\vec{x}} + \gamma^\dagger(\vec{q},r) \frac{v(\vec{q},r)}{\sqrt{2q}} e^{-iq/Ha - i\vec{q}\cdot\vec{x}} \right\}, \tag{4.7}$$

$$\overline{\psi}_I(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3} \sum_s \left\{ \gamma(\vec{k},s) \frac{\overline{v}(\vec{k},s)}{\sqrt{2k}} e^{ik/Ha + i\vec{k}\cdot\vec{x}} + \beta^\dagger(\vec{k},s) \frac{\overline{u}(\vec{k},s)}{\sqrt{2k}} e^{-ik/Ha - i\vec{k}\cdot\vec{x}} \right\}. \tag{4.8}$$

Note that we have absorbed a factor of $a^{\frac{3}{2}}$ into ψ_I and $\overline{\psi}_I$ so that they agree exactly with the flat space expansions in conformal coordinates. The spinor wave functions obey the usual spin sum formulae of massless fermions in flat space,

$$\sum_s u(\vec{k},s)\overline{u}(\vec{k},s) = \not{k} = \sum_s v(\vec{k},s)\overline{v}(\vec{k},s) \,. \tag{4.9}$$

The nonzero relations of the operator algebra are,

$$\Big[\alpha(\vec{p}), \alpha^\dagger(\vec{p}\,')\Big] = (2\pi)^3 \delta^3(\vec{p} - \vec{p}\,') \,, \tag{4.10}$$

$$\Big\{\beta(\vec{k},s), \beta^\dagger(\vec{k}',s')\Big\} = (2\pi)^3 \delta^3(\vec{k}-\vec{k}')\delta_{ss'} = \Big\{\gamma(\vec{k},s), \gamma^\dagger(\vec{k}',s')\Big\} \,. \tag{4.11}$$

We are studying the same Yukawa theory as in sections 2 and 3 and, as before, we release the universe in free Bunch-Davies vacuum at $t = 0$,

$$\alpha(\vec{p})|\Omega\rangle = 0 = \beta(\vec{k},s)|\Omega\rangle = \gamma(\vec{q},r)|\Omega\rangle \,. \tag{4.12}$$

Therefore the operator which maps the various free fields into interacting fields is,

$$\mathrm{U}(t) = T\left\{ \exp\Big[-if\int_0^t dt' \int d^3x \phi_I(t',\vec{x}) \overline{\psi}_I(t',\vec{x}) \psi_I(t',\vec{x})\Big] \right\}. \tag{4.13}$$

Although Heisenberg states such as $|\Omega\rangle$ do not change with time, what these states mean is determined by operators, which do change with time. In particular, the mapping,

$$a^{\frac{3}{2}}(t)\psi(t,\vec{x}) = U^\dagger(t)\psi_I(t,\vec{x})\,\mathrm{U}(t) \,, \tag{4.14}$$

suggests a reasonable definition for the operator which annihilates fermions at time t,

$$\beta(t,\vec{k},s) \equiv U^\dagger(t)\beta(\vec{k},s)\,\mathrm{U}(t) \,. \tag{4.15}$$

We therefore arrive at an expression for the probability that $|\Omega\rangle$ contains a fermion of wave number $\vec{k}$ and spin s at time t,

$$P_\beta(t,\vec{k},s)(2\pi)^3\delta^3(\vec{k}-\vec{q})\delta_{rs} = \Big\langle \Omega \Big| \beta^\dagger(t,\vec{q},r)\beta(t,\vec{k},s) \Big| \Omega \Big\rangle = \Big\langle \Omega \Big| U^\dagger(t)\beta^\dagger(\vec{q},r)\beta(\vec{k},s)\,\mathrm{U}(t) \Big| \Omega \Big\rangle \,. \tag{4.16}$$

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Expression (4.16) is straightforward to evaluate to lowest order. One first commutes the creation and annihilation operators through the factors of $U^{\dagger}(t)$ and $\mathrm{U}(t)$,

$$P_{\beta}(t,\vec{k},s)(2\pi)^3\delta^3(\vec{k}-\vec{q})\delta_{rs} = \Bigl\langle \Omega \Bigl| \Bigl[\beta(\vec{q},r),\mathrm{U}(t)\Bigr]^{\dagger} \Bigl[\beta(\vec{k},s),\mathrm{U}(t)\Bigr] \Bigr| \Omega \Bigr\rangle . \tag{4.17}$$

From the definition (4.13) of $\mathrm{U}(t)$, and the free field expansions, we find,

$$\Bigl[\beta(\vec{k},s),\mathrm{U}(t)\Bigr] = \frac{-if}{\sqrt{2k}} \int_0^t dt' \int d^3x \phi_I(t',\vec{x}) \overline{u}(\vec{k},s) \psi(t',\vec{x}) e^{-ik/Ha' - i\vec{k}\cdot\vec{x}} + O(f^2) . \tag{4.18}$$

The free field expansions imply,

$$\begin{aligned} \Bigl\langle \Omega \Bigl| \phi_I(t'',\vec{y}) \overline{\psi}_I(t'',\vec{y}) u(\vec{q},r) \phi_I(t',\vec{x}) \overline{u}(\vec{k},s) \psi_I(t',\vec{x}) \Bigr| \Omega \Bigr\rangle = \\ = \int \frac{d^3p}{(2\pi)^3} f^*(t',p) f(t'',p) e^{-i\vec{p}\cdot(\vec{x}-\vec{y})} \times \\ \times \int \frac{d^3\ell}{(2\pi)^3} \frac{1}{2\ell} \sum_{s'} \overline{v}(\vec{\ell},s') u(\vec{q},r) \overline{u}(\vec{k},s) v(\vec{\ell},s') e^{-i\ell/Ha' + i\ell/Ha'' - i\vec{\ell}\cdot(\vec{x}-\vec{y})} , \end{aligned} \tag{4.19}$$

The integrals over $\vec{x}$ and $\vec{y}$ enforce $\vec{q} = \vec{k}$ and $\vec{\ell} = -(\vec{k}+\vec{p})$. With these identifications we can set $r = s$ and spin average to eliminate the fermion wave functions,

$$\frac{1}{2} \sum_{s,s'} \overline{v}(\vec{\ell},s') u(\vec{k},s) \overline{u}(\vec{k},s) v(\vec{\ell},s') = \frac{1}{2} \mathrm{Tr}\Bigl[\not{\ell} \not{k} \Bigr] = 2 \Bigl[\|\vec{k}+\vec{p}\| k + (\vec{k}+\vec{p})\cdot\vec{k} \Bigr] . \tag{4.20}$$

Putting everything together gives,

$$\begin{aligned} P_{\beta}(t,\vec{k},s) = \frac{f^2}{2k} \int_0^t dt' \int_0^t dt'' \int \frac{d^3p}{(2\pi)^3} f^*(t',p) f(t'',p) e^{-i(\frac{1}{Ha'} - \frac{1}{Ha''})(\|\vec{p}+\vec{k}\|+k)} \times \\ \times \Biggl[k + \frac{(\vec{k}+\vec{p})\cdot\vec{k}}{\|\vec{k}+\vec{p}\|} \Biggr] + O(f^4) . \end{aligned} \tag{4.21}$$

The time dependence of the integrand of (4.21),

$$f^*(t',p) f(t'',p) e^{-i(\frac{1}{Ha'} - \frac{1}{Ha''})(\|\vec{p}+\vec{k}\|+k)} = \frac{H^2}{2p^3} \Bigl(1 + \frac{ip}{Ha'}\Bigr) \Bigl(1 - \frac{ip}{Ha''}\Bigr) e^{-i(\frac{1}{Ha'} - \frac{1}{Ha''})(\|\vec{p}+\vec{k}\|+p+k)} , \tag{4.22}$$

indicates that the lowest order effect is driven by the physical process in which a scalar and a fermion-anti-fermion pair emerge from the vacuum. In flat space quantum field theory this process would be forbidden by energy conservation. It is crucial to note that this is not put in by hand: even in flat space one gets an expression analogous to (4.21), but it goes to zero as the time t tends to infinity. What drives the flat space expression to zero is the fact that no value of the scalar wave vector $\vec{p}$ gives zero for the total flat space energy, $\|\vec{p}+\vec{k}\| + p + k$. Hence the temporal integrations result in rapid oscillations that tend to cancel. As t grows, the result approaches an energy delta function that cannot be saturated. In our locally de Sitter background the inverse scale factors redshift the physical energy to zero so rapidly that there are only a finite number of oscillations.

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Note the essential role played by the breaking of conformal invariance. Since free fermions are conformally invariant, and the Yukawa interaction is not inconsistent with conformal invariance, the effect of conformal breaking is concentrated in the scalar wave function, $f(t,p)$. Had we instead used a conformally coupled scalar the integrand (4.22) would go to,

$$f^*(t',p)f(t'',p)e^{-i(\frac{1}{Ha'}-\frac{1}{Ha''})(\|\vec{p}+\vec{k}\|+k)} \longrightarrow \frac{1}{2a'a''p}e^{-i(\frac{1}{Ha'}-\frac{1}{Ha''})(\|\vec{p}+\vec{k}\|+p+k)}\,, \tag{4.23}$$

In this case the inverse scale factors suppress the result.

To give a good approximate computation of $P_\beta(t,\vec{k},s)$ we note, from the preceding discussion, that scalar wave numbers with $p \gtrsim HA \equiv H\min(a',a'')$ engender oscillations in the energy exponent. We can therefore cut the scalar wave number off above this limit. (The infrared cutoff of $p \geq H$ derives from the finite coordinate range of our spatial manifold T^3 [9].) The same energy exponent also gives rise to oscillations when $k \gtrsim Ha'$ or $k \gtrsim Ha''$, which cuts off the lower limits of the temporal integrations at $t',t'' \geq T = \ln(w)/H$, where $w \equiv k/H$. With these cutoffs in place we can neglect terms that fall off like $1/a'$ or $1/a''$,

$$P_\beta(t,\vec{k},s) \longrightarrow \frac{f^2}{k}\int_T^t dt' \int_T^t dt'' \int_H^{HA} dp\, p^2 \frac{H^2}{2p^3} \int \frac{d^2\widehat{p}}{(2\pi)^3}\left[k + \frac{(\vec{k}+\vec{p})\cdot\vec{k}}{\|\vec{k}+\vec{p}\|}\right] + O(f^4)\,. \tag{4.24}$$

The angular integration is straightforward,

$$\int \frac{d^2\widehat{p}}{(2\pi)^3}\left[k + \frac{(\vec{k}+\vec{p})\cdot\vec{k}}{\|\vec{k}+\vec{p}\|}\right] = \frac{k}{2\pi^2}\left\{\theta(k-p)\left[2-\frac{p^2}{3k^2}\right] + \theta(p-k)\left[1+\frac{2k}{3p}\right]\right\}. \tag{4.25}$$

Now change variables from time to e-foldings, and take advantage of the symmetry between t' and t'',

$$\begin{aligned} P_\beta(t,\vec{k},s) \longrightarrow \frac{f^2}{2\pi^2}\int_{\ln w}^N dN' \int_{\ln w}^{N'} dN'' \int_H^{He^{N''}} \frac{dp}{p} \times \\ \times \left\{\theta(k-p)\left[2-\frac{p^2}{3k^2}\right] + \theta(p-k)\left[1+\frac{2k}{3p}\right]\right\} + O(f^4)\,. \end{aligned} \tag{4.26}$$

Performing the integrations and retaining only the leading terms for $N \gg \ln(w) \gg 1$, we obtain,

$$P_\beta(t,\vec{k},s) \longrightarrow \frac{f^2}{2\pi^2}\int_{\ln w}^N dN' \int_{\ln w}^{N'} dN'' \Big\{N'' + \ln(w) + \frac{1}{2} + \ldots\Big\} + O(f^4)\,, \tag{4.27}$$

$$\longrightarrow \frac{f^2}{12\pi^2}\Big\{N^3 + \ldots\Big\} + O(f^4)\,. \tag{4.28}$$

As one might have expected, our leading order result for $P_\beta(t,\vec{k},s)$ is independent of s and w. Upon taking a square root of $P_\beta(t,\vec{k},s)$, one gets a quite good approximation for the exponent of the WKB mode function (3.17) we found in section 3.

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5. Discussion

We have found that the effect of inflation on Yukawa-coupled, massless fermions is in some ways opposite to the effect on the photons of massless scalar QED. Whereas the photon becomes massive, but suffers no significant particle production [7], we find that fermions remain massless but experience enormous particle production. The physical mechanism is simple: inflation alters the constraint of energy conservation so that three massless particles can spontaneously emerge from the vacuum. The Yukawa interaction mediates this for a scalar and a fermion-anti-fermion pair, and the process has an appreciable amplitude when the scalar is minimally coupled.

The mathematical analysis for Yukawa-coupled fermions is quite different from that required for photons in scalar QED. Although the mode equations of both systems are nonlocal, the integrals of the fermion system are dominated by their upper limits because the fermion mode function grows. This means that we need only drop exponentially suppressed terms and then differentiate to extract a local equation for the asymptotic form. The photon mode equations were much more subtle because the photon mode function approaches a nonzero constant — as do classical photons — and the asymptotic effect of vacuum polarization shows up in the increasingly rapid oscillations around this value.

Of course fermions obey the Pauli exclusion principle, so at most one particle can be put in each momentum and spin state. The amplitude of the wave function cannot really grow past that point. Therefore the rate of production for any particular momentum and spin state must go to zero as the probability of this state being occupied approaches unity. This phenomenon is known as *Pauli blocking*. We do not see Pauli blocking because we are using only the one loop self energy. We expect Pauli blocking to appear at two loop order.

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Erratum

Abstract. We correct a sign error in the effective Dirac equation that was solved in [12]. We discuss the changes this implies and their physical significance.

In ref. [12] we calculated the one-loop self-energy for massless fermions which couple *via* a Yukawa interaction term to a massless, minimally coupled scalar during de Sitter inflation. However, within the system of conventions we used, the self-energy should contribute oppositely to the local derivative term in the quantum-corrected Dirac equation. The correct equation is,

$$a^{3/2} i\partial\!\!\!/ (a^{3/2}\psi(x)) - \int d^4x' \Sigma_{\rm ret}(x,x')\psi(x') = 0\,. \tag{5.1}$$

Here $a = -1/(H\eta)$ denotes the scale factor during de Sitter inflation, $\Sigma_{\rm ret}$ is the retarded self-energy of the Schwinger-Keldysh formalism, and ψ is the fermionic wave function. This is how equation (29) in [12] should have read.

The correct sign makes a simple and profound change in our analysis. The conformally rescaled, chiral components of spatial plane wave mode functions obey the equation,

$$\partial_N h^{\pm}_L(N,w) + \Bigl(\frac{f}{2\pi}\Bigr)^2 \int_0^N dN'\, h^{\pm}_L(N',w) e^{\mp i 2we^{-N'}} \Biggl[N' - \frac{3}{4} - \frac{1}{2}\int_0^{2we^{-N'}} d\tau\, \frac{e^{\pm i\tau}-1}{\tau}\Biggr] \approx 0\,. \tag{5.2}$$

where $w = k/H$, $N = \ln(a)$, f denotes the Yukawa coupling, and the approximation is that some exponentially small terms have been neglected. This should replace equation (38) in [12]. In the late time limit of large $N = \ln(a)$, both helicity components approach the same declining oscillatory form,

$$h^{\pm}_{L,R} \stackrel{N\rightarrow\infty}{\longrightarrow} \Bigl(N - \frac{3}{4}\Bigr)^{-1/4} \exp\Biggl(\pm {\rm i}\frac{f}{3\pi}\Bigl(N-\frac{3}{4}\Bigr)^{3/2}\Biggr)\,. \tag{5.3}$$

This replaces equation (41) in [12].

The correct asymptotic solution (5.3) stands in sharp contrast to the faster-than-exponential growth which pertains with the sign error! The physical interpretation of the corrected result seems to be that the inflationary production of massless, minimally coupled scalars drives the scalar field strength away from zero, which affects super-horizon fermions like a mass term. For a more complete study of the mass generation mechanism in the case when the scalar field is light and nearly minimally coupled we refer to ref. [13].

Declining oscillatory behavior very similar to (5.3) is observed when studying late time dynamics of super-Hubble photons [14]. So the sign correction brings our results for Yukawa into pleasant conformity with what charged scalars do to photons during inflation [15, 16]. In each case inflationary particle production forces the scalar away from zero, whereupon the effect on other fields is qualitatively similar (but not in every detail!) to what happens for nonzero scalar backgrounds in symmetry breaking.

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Note that the sign change reported here does not affect either the one loop Yukawa scalar self-mass-squared or its impact on plane wave scalar mode functions [17]. So the conclusion of that study is still that the scalar cannot develop a large enough mass rapidly enough to affect what it does to the fermion. Nor does the sign change reported here affect the recent computation of quantum gravitational corrections to the one loop self-energy of massless fermions in Dirac + Einstein [18]. Of course our sign change very much *does* affect the impact these corrections have on fermion mode functions [19]!

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